observed cross section; i.e., $E_i = E_b + \Delta E$. We find the latter view the more useful and opted to calculate amounts of energy ΔE which should be added to E_b at each point to correct for the spread of the beam. For use in making this correction we calculated a fine-meshed table of ΔE, part of which is reproduced as Table VI. The entries therein are given as values of $\Delta E/W_{1/2}$. For energies higher than the range of the table, a reasonably accurate approximation to use is

$$\frac{\Delta E}{W_{1/2}} \propto \left(\frac{E_b - E_{th}}{W_{1/2}}\right)^{-1}.$$

Our specific calculational procedure was as follows: (1) From the raw data (or, better, using reasonable guesses for values of ΔE) obtain preliminary values of $b(0)$ and E_{th} for a given value of a; (2) using the two data points and this first approximation to E_{th} calculate m; (3) using m and the known values of $W_{1/2}$ for each data point obtain the corresponding values of ΔE from the table; (4) using these values of ΔE to correct the energies E_b associated with the data points, go back and calculate improved values of $b(0)$ and E_{th}, and so on. We showed that our procedure converges rapidly to the same point from both directions.

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Differential Range Study of Products Formed by 2.9-GeV Proton Irradiation of Silver*

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Thin silver targets (≈0.1 mg/cm²) were irradiated with 2.9-GeV protons. The recoiling products were collected in a stack of thin plastic films at 90° to the beam, at a geometry of 2%. By radiochemical separation Sr^{83}, $Cu^{61,64}$, $Sc^{43,44}$, $K^{42,43}$, and Na^{24} were removed from the films. Range distributions were obtained for each, and from these the corresponding energy spectra and velocity spectra were deduced. The mean energies are: Sr^{83}—3.2 MeV, $Cu^{61,64}$—5.9 MeV, $Sc^{43,44}$—9.4 MeV, Na^{24}—18.1 MeV. Comparison of the observed spectra with the results of Monte Carlo cascade-evaporation calculations showed excellent agreement for all of the products except Na^{24}. It is concluded that the former are mainly spallation residues while Na^{24} is formed by the splitting of a parent nucleus into two fragments. All of the energy spectra are very broad and there is no sharp difference between "spallation-type" spectra and "two-body breakup" type spectra.

INTRODUCTION

THERE has been extensive discussion in the literature[1–4] about the mechanisms involved in high-energy nuclear reactions. For the production of species not too far removed from medium weight targets it is generally agreed that a two-step, cascade-evaporation, process predominates. Monte Carlo calculations based on this model[5,6] give mass-yield curves which are in fair agreement with experiment down to products whose mass is about half that of the target nuclei. Species substantially lighter than half the target mass are most probably produced by some other mechanisms, those involving break up into two bodies. This has often been called[2–4,7,8] "fragmentation." It is thought to occur at a time very close to that of the fast nucleon cascade. The term "fission" used for heavy elements has also been applied[8–12] to a two body breakup of medium weight elements, but then one usually thinks of a longer time scale, comparable with evaporation times. Numerous experiments[2–4,7] have been interpreted by means of the fragmentation mechanism. In a recent nuclear emulsion investigation[12] evidence was presented for a mechanism more properly called "fission."

In the present work, recoil range distributions were obtained for a series of products of widely differing

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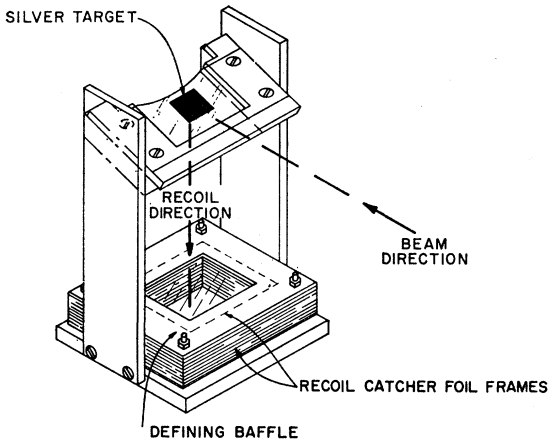

FIG. 1. Apparatus for measuring range distributions at 90° to beam direction.

masses from silver targets irradiated by 2.9-GeV protons. It was thought that Sr^{83} would represent a pure spallation product and that Na^{24} would represent a product formed solely by two body breakup. $Cu^{61,64}$ may also be primarily a spallation residue while $Sc^{43,44}$ might be produced in comparable amounts by both of these two possible mechanisms. The earlier nuclear emulsion experiments[11,12] suggested that there are substantial differences in range distributions between spallation products and "fission" products; perhaps the formation of a given product by a spallation mechanism would give rise to shorter ranges than formation of the same product by a two-body breakup mechanism. If this were true, one might be able, for certain products of intermediate mass, to resolve two groups by differential range measurements. As we shall see below, no such resolution was found for any of the products studied. However, comparison of our results with a calculation based on the spallation mechanism gave an indication of the mass regions where each of the two mechanisms predominate.

EXPERIMENTAL

The target assembly shown in Fig. 1 was used to measure the range distributions of radiochemically selected recoil produced by the interaction of 2.9-GeV protons with silver. The target consisted of a 1-cm square of silver, vacuum deposited on a thin plastic backing. Several target and backing thicknesses, as indicated in Table I, were used to investigate the effects of scattering on the observed range distributions. The target foils were inclined at an angle of 30° to the beam direction so that path lengths of the recoils in the silver would be less than 15% longer than the normal target thickness.

The catcher foils were prepared from thin plastic and were mounted on aluminum frames having 2-in. square holes. To ensure that all foils subtended the same solid angle, an aluminum baffle with a $1\frac{1}{2}$-in. square hole was placed over the catcher foil stack. Both baffle and foil frames were coated with plastic to prevent contamination of the foils by recoils from the aluminum. The target to baffle distance was 3 in. and the solid angle subtended was $\approx 2\%$ of 4π sr.

Foils for the first five irradiations were fabricated from Liquidope (cellulose nitrate), but considerable difficulty was experienced in separating them from each other after the irradiations. In the last two experiments Formvar (polyvinyl formal) foils were used which were readily separated. These films were prepared by spreading a cyclohexanone solution of Formvar on a large water surface. The thickness of the catcher foils was varied from ≈ 70 to ≈ 400 $\mu g/cm^2$ depending on the resolution desired. The thickness and uniformity of the foils were measured with an alpha thickness gauge.[13] Twenty foils were used in each irradiation. As the experiment progressed, increasing care was taken to insure uniform foils. In the later irradiations, the thickness of each foil was measured at five points by two observers, and only those foils which were uniform to better than $\approx 10\%$ in thickness were used.

Seven irradiations were performed, as indicated in Table I, with total circulating beams up to $\approx 2\times 10^{15}$ protons. During the irradiations, a massive copper target was located in another straight section of the Cosmotron. This cast a "shadow" on the target assembly and prevented the primary beam from striking the catcher foils. A movable shutter in still another straight section prevented low-energy particles from hitting the silver target. For the last two irradiations listed in Table I, the Cosmotron magnet was operated in the "flat-topped" mode.[14] This resulted in a substantial increase in the number of proton traversals through the target compared to the earlier irradiations. Hence, the activities of the samples from the

TABLE I. Irradiation data.

Irradiation	Ag target thickness $\mu g/cm^2$	Plastic backing $\mu g/cm^2$	Beam intensity $\times 10^{-15}$	Recoils analyzed
1	200	200	0.15	K, Cu
2	200	200	0.56	K, Cu
3	200	200	1.48	Na, Sc
4	106	177	2.19	Cu, Sr
5	108	115	1.0	Cu, Sr
6[a]	31	100	1.02	Cu, Sr
7[a]	56	100	1.87	Na, Sc

[a] Best irradiations: thinnest targets and Formvar catchers. Irradiations 1–5 had Liquidope catchers.

[13] K. Ramavataram and D. I. Porat, Nucl. Instr. Methods 4, 239 (1959).

[14] The application of "flat topping" for recoil experiments was first proposed and tested by L. P. Remsberg. The rate of change of the magnetic field with time is greatly reduced at the end of the acceleration cycle thus increasing the number of proton traversals through very thin targets.

irradiations with the thinner targets were comparable to those with the thicker ones.

After irradiation, the target assembly was taken apart and the individual foils were carefully separated from their frames and transferred to centrifuge cones containing 5–10 mg of carriers of the elements of interest. The foils were dissolved and the organic matter destroyed with aqua regia. The elements separated were Sr, Cu, Sc, K, and Na. Because of the substantial variation of range between Sr and Na and because of the number of samples involved, only two elements, as shown in Table I, were separated in any irradiation. The chemical procedures used were modified from standard radiochemical methods. The principal steps are summarized below.

Strontium: Passage through a Dowex-1 anion exchange column in 9 *M* HCl, three precipitations of $Sr(NO_3)_2$ with 80% HNO_3, $Fe(OH)_3$ scavenging, and precipitation as $SrCO_3$.

Copper: Adsorption on a Dowex-1 column from 9 *M* HCl, elution by 3 *M* HCl, precipitation as CuS, fuming with HBr, $Fe(OH)_3$ scavenging, and precipitation as CuCNS.

Scandium: Passage through a Dowex-1 column in 9 *M* HCl, extraction from 9 *M* HCl by tributyl phosphate (TBP), extraction from 0.1 *M* HCl by thenoyl trifluoroacetone (TTA), and precipitation as ScF_3.

Potassium: Passage through a Dowex-1 column in 9 *M* HCl, scavenging precipitations of rubidium with alcoholic chlorostannic acid, scavenging with $Fe(OH)_3$ and $SrCO_3$, and finally three precipitations with sodium tetraphenylboron. Unfortunately, decontamination of potassium from the much greater rubidium activities in the first foils was not adequate and, after two irradiations, potassium was replaced by scandium. However, as will be seen below, the potassium and scandium range distributions agree well, beyond the first two foils.

Sodium: The aqueous phase from the first scandium extraction was evaporated to small volume and sodium was precipitated with zinc uranylacetate reagent. The precipitate was converted to the chloride with HCl-butanol solution, dissolved in 9 *M* HCl, and passed through a Dowex-1 column. The solution was scavenged with $Fe(OH)_3$ and ammonium salts removed by fuming. Potassium was separated as the perchlorate, and NaCl was precipitated for the final sample.

After the pure chemical fractions were obtained they were mounted for activity measurements with end-window flow proportional counters. Anticoincidence shielding was used to reduce the counter backgrounds to $\approx$0.3 counts/min for the assay of some of the samples having low activities. The normal and low-background counters were intercalibrated by counting at least one sample of each element on both types of counters. The efficiencies of the low-background counters were between 10 and 20% less than those of the standard counters depending on the particular isotope used for the calibration. It was assumed that the variation of counting efficiency with sample thickness could be ignored for the range of thicknesses and the isotopes of interest in this experiment.

The decay curves of the samples were analyzed by a least-squares procedure to give the activities of the various components. In the case of strontium samples, a preliminary Sr–Y separation was performed a few hours after the end of the irradiation. However, final purification and activity measurements did not take place until the day after the irradiation; hence, no short lived strontium activities are expected. The decay curves resolved well into components due to Sr^{83}($t_{1/2}$=32.4 h)[15] and to Sr^{82}($t_{1/2}$=25 day). The latter accounted for $\approx$25% of the total activity at the start of counting.

The copper samples contained Cu^{61}($t_{1/2}$=3.3 h), Cu^{64}($t_{1/2}$=12.8 h), and Cu^{67}($t_{1/2}$=61 h). Because of the low levels of activity, there was some uncertainty in the resolution of the decay curves. The activity of Cu^{67} was only 2–3 counts/min at the most and therefore no results are reported below for this isotope. However, within the substantial errors, no significant differences in the range curves of Cu^{61}, Cu^{64}, or Cu^{67} were observed; hence we have reported a composite $Cu^{61,64}$ value taken from the best fit decay curves. This procedure was adopted since it was observed that the composite $Cu^{61,64}$ activities, derived from the sums of Cu^{61} and Cu^{64} activities in the samples at a given time, showed a much more regular behavior, i.e., a smoother range curve, than either those of Cu^{61} or Cu^{64} alone.

The scandium decay curves were resolved into a 4.0-h component (the sum of Sc^{43} and Sc^{44}) and a longer lived tail. This was approximated by a $\approx$3-day half-life although it is undoubtedly a mixture of several scandium activities. The tail amounted to $\approx$50% of the 4-h activity at the start of counting.

In potassium samples, a composite of K^{42}($t_{1/2}$=12.5 h) and K^{43}($t_{1/2}$=22 h) was obtained. As mentioned above, rubidium contamination was present in the first two foils.

The activity in the sodium samples decayed with the 15.0-h half-life of Na^{24} and a small amount ($\approx$5%) of longer lived component.

In all cases, samples were separated from foils beyond the range of the isotopes of interest. These showed that activation of impurities in the foils, or recoils from the foil frames, to give strontium, copper, scandium, and potassium activities was negligible. For Na^{24} there was a small blank which was subtracted from the results reported below.

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[15] I. Dostrovsky, S. Katcoff, and R. W. Stoenner, J. Inorg. Nucl. Chem. **26**, 209 (1964).

TABLE II. Data for calculating range, energy, and velocity distributions at 90° of $Cu^{61,64}$ and of Sr^{83} measured in irradiation 6. The corresponding range, energy, and velocity spectra were derived by plotting the activities per unit thickness, energy, or velocity, respectively, for each interval.

		$Cu^{61,64}$			Sr^{83}		
Catcher foil	Foil thickness (mg/cm²)	Mean energy (MeV)	Mean velocity (100 β)	Activity (corr counts/min)	Mean energy (MeV)	Mean velocity (100 β)	Activity (corr counts/min)
1	0.0718	0.3	0.29	31.8	0.4	0.29	72.8
2	0.0718	0.8	0.52	23.0	1.1	0.51	71.1
3	0.0722	1.4	0.68	28.2	1.7	0.66	71.5
4	0.0726	1.9	0.81	33.8	2.4	0.78	67.2
5	0.0737	2.6	0.94	34.9	3.1	0.90	65.9
6	0.0745	3.4	1.07	55.4	4.0	1.01	53.9
7	0.0718	4.2	1.18	58.9	4.8	1.11	43.3
8	0.0741	5.0	1.30	64.1	5.7	1.21	33.2
9	0.0749	6.0	1.42	64.3	6.6	1.31	26.4
10	0.0741	7.0	1.53	58.9	7.6	1.40	16.5
11	0.0741	8.1	1.65	55.4	8.7	1.50	11.8
12	0.0756	9.2	1.76	42.0	9.8	1.59	7.4
13–14	0.1516	11.1	1.93	58.2	11.8	1.74	5.4
15–16	0.1547	13.7	2.14	26.3	14.7	1.94	1.0
17–18	0.1574	16.5	2.35	9.4			
19–20	0.1569	19.8	2.58	2.3			

RESULTS

The differential range data are summarized in Tables II and III and in Fig. 2. In each case data from the run with the thinnest silver target are presented (see Table I). Duplicate runs, but with thicker Ag targets, gave very good agreement except that the first and second plastic catcher foils showed somewhat higher activities. For example, when the target was 200 μg/cm² rather than 56 μg/cm² the normalized Sc activity was 70% higher in the first foil and 35% higher in the second foil. Evidently there was appreciable backscattering of recoils in the thicker targets. However, no corrections were made for the small residual effect in the thinner targets since it was felt that only the very low energy portions of the spectra are involved. In the case of $K^{42,43}$ there was appreciable Rb contamination in the potassium fraction separated from the first two films. Beyond the second film (0.4 mg/cm²) the range distributions of $K^{42,43}$ and $Sc^{43,44}$ are closely similar (Fig. 2).

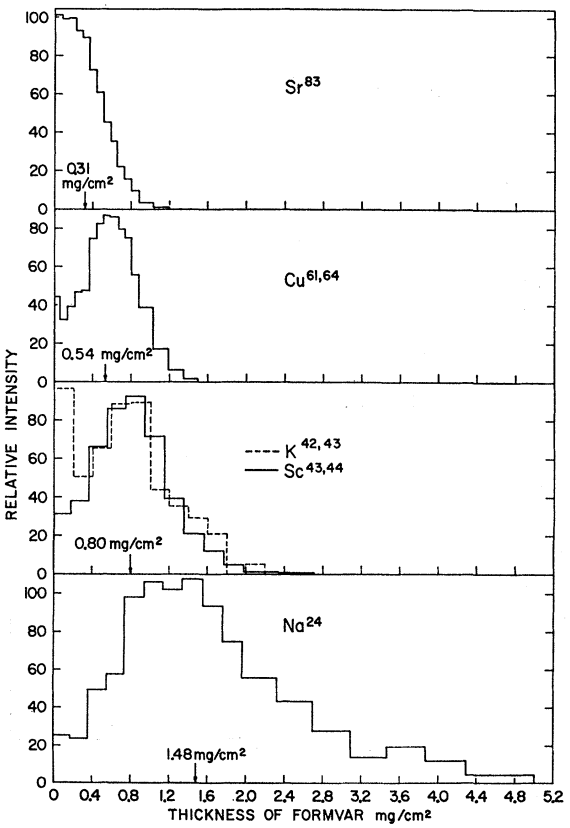

FIG. 2. Observed range distributions at 90° of products from 2.9-GeV proton irradiation of silver.

To convert the range distributions to energy spectra it is necessary to have range-energy relationships for each of the species investigated. Unfortunately, there are very few experimental data on the stopping of heavy atoms in plastic foils for the energy region of interest. Therefore, the theoretical calculations of Linhard, Scharff, and Schiott[16] were used as a guide to derive the range-energy relationships shown in Fig. 3. It was assumed that the stopping power of Formvar and Liquidope are the same and are approximated by that of beryllium. In the case of Na^{24} the curve above 17 MeV was adjusted to be consistent with Northcliffe's data[17] on the range of Ne^{20} ions in aluminum. It is estimated that the energies of recoil atoms derived from Fig. 3 may be uncertain by as much as 20%.

Energy spectra of the various species from proton irradiation of silver are shown in Fig. 4 as the smooth

[16] J. Linhard, M. Scharff, and H. E. Schiott, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **33**, No. 14 (1963).
[17] L. C. Northcliffe, Phys. Rev. **120**, 1744 (1960).

TABLE III. Data for calculating range, energy, and velocity distributions at 90° of Na^{24} and of $Sc^{43,44}$ measured in irradiation 7. The corresponding range, energy, and velocity spectra were derived by plotting the activities per unit thickness, energy, or velocity, respectively, for each interval.

		Na^{24}			$Sc^{43,44}$		
Catcher foil	Foil thickness (mg/cm^2)	Mean energy (MeV)	Mean velocity (100 β)	Activity (corr counts/min)	Mean energy (MeV)	Mean velocity (100 β)	Activity (corr counts/min)
1	0.184	0.4	0.42	46	0.7	0.44	287
2	0.187	1.5	1.03	43	2.1	0.97	353
3	0.188	2.9	1.57	91	3.8	1.36	626
4	0.192	4.9	2.09	109	6.0	1.72	822
5	0.198	7.5	2.59	192	8.7	2.09	911
6	0.201	10.6	3.03	212	12.1	2.48	715
7	0.202	13.5	3.48	204	16.7	2.89	392
8	0.207	15.9	3.72	221	22.4	3.32	217
9	0.207	18.3	4.00	191	28.7	3.74	123
10	0.208	20.6	4.26	155	35.0	4.12	47
11	0.365	23.9	4.59	203	43.8	4.62	19
12	0.382	28.2	4.99	163	54.7	5.23	6
13	0.385	32.7	5.37	106			
14	0.387	36.9	5.72	52			
15	0.392	40.9	6.07	75			
16	0.403	45.1	6.33	47			
17–18	0.770	51.5	6.73	31			
19–20	0.761	59.0	7.25	0			

curves drawn through the experimental points. These spectra have been obtained from the data presented in Tables II and III. A small correction was made in each case for energy loss within each target. The spectra show a steady shift to higher energy as the mass of the product is decreased. No two range groups are seen for any of the species. All of the spectra, except the one for Sr^{83}, show most probable energies well above zero. Mean energies are listed in Table IV and are indicated in Fig. 4 by solid arrows surmounted by an *M*.

TABLE IV. Mean ranges and energies of products at 90° from 2.9-GeV proton irradiation of silver.

Species	Mean range in Formvar (mg/cm^2)	Mean energy (obs) (MeV)	Mean energy (calc)[a] (MeV)
Na^{24}	1.48	18.1	12.3
$Sc^{43,44}$	0.80	9.4	9.9
$Cu^{61,64}$	0.54	5.9	7.9
Sr^{83}	0.31	3.2	4.3

[a] Cascade-evaporation calculation.

DISCUSSION

The energy spectra may be compared with the results of cascade-evaporation calculations. We start with the results of the Monte Carlo cascade calculations of Metropolis *et al.*[5] for Ru^{100} irradiated by 1.8-GeV protons. The residual nuclei predicted by this calculation were first appropriately shifted in Z and A and were then used as the starting nuclei for an evaporation calculation. The components of the velocity imparted to these residual nuclei by the cascade were taken from the calculation of Porile.[18] The velocity values were reduced by the ratio of the masses of the residual nuclei before the above mentioned shift to those after the shift in A. This is consistent with the lack of dependence of the calculated momentum-excitation energy relationship[18] on target mass number. It has been pointed out[19] that the calculation of Porile tends to overestimate the transverse component of momentum of the residual nuclei. This is due to the random choice of sign of the component of momentum of the cascade particles along one of the axes perpendicular to the beam direction. The sign of this quantity had not been recorded in the calculation of Metropolis *et al.*[5] The magnitude of the error introduced by this procedure was estimated in the manner discussed by Porile and Tanaka[19] and the velocity component of the residual nuclei along the appropriate transverse axis was reduced by 40%.

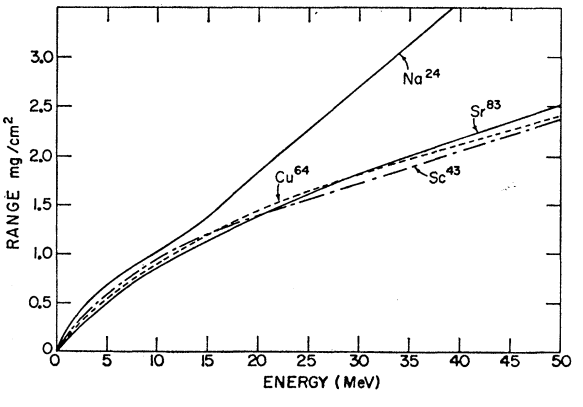

FIG. 3. Range versus energy relationships used to convert range distributions to spectra.

[18] N. T. Porile, Phys. Rev. **120**, 572 (1960).

[19] N. T. Porile and S. Tanaka, Phys. Rev. (to be published).

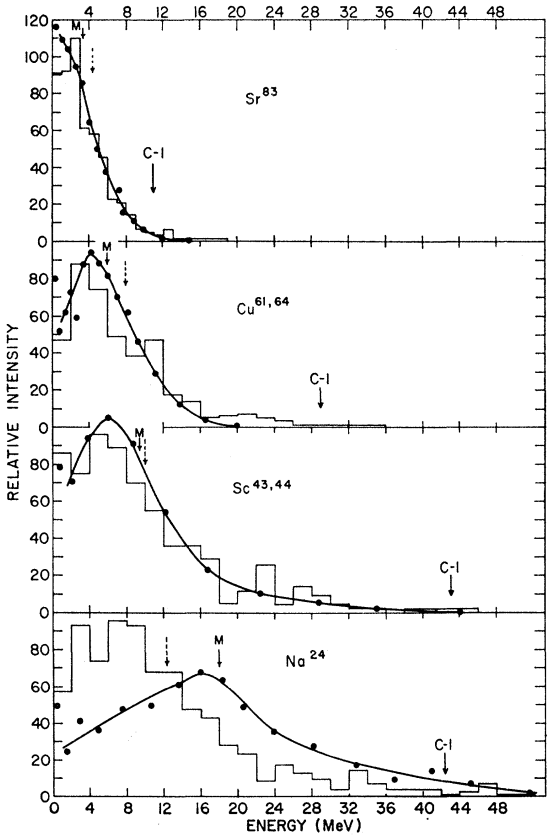

FIG. 4. Energy spectra at 90° of products from 2.9-GeV proton irradiation of silver. Experimental data are shown by the points and the smooth curves. The histograms show the results from the cascade-evaporation calculations. *M* indicates experimental mean energy; dashed arrow indicates mean energy calculated for cascade-evaporation mechanism; C-1 indicates limiting value of Coulomb energy calculated for split into two fragments.

The shifted residual nuclei with their corrected velocity components were used as the starting nuclei for a Monte Carlo evaporation calculation. The calculation was based on the treatment due to Dostrovsky *et al.*[20] and has been described in detail elsewhere.[19] The calculation was performed on an IBM 7094 computer. In order to improve the statistical accuracy, 10 evaporation calculations were performed for each of the approximately 400 starting nuclei on the assumption that the target was Ag^{107}, and 10 calculations were performed assuming an Ag^{109} target. The level density parameter a was set equal to $A/10$, r_0 was taken as 1.5×10^{-13} cm and the pairing energies were obtained from Cameron.[21]

The evaporation calculation normally considers the emission of the 6 lightest particles through He^4. In the present calculation the evaporation of heavier particles was included since recent studies of the interaction of high-energy protons with heavy emulsion nuclei[11] indicate that the probability for the emission of fragments with $Z=3$–6 is about 0.3 per interaction. The calculation of the emission probabilities of heavy particles followed the treatment of Dostrovsky *et al.*[22] Since these particles may be emitted in bound excited states as well as in the ground state, a large number of separate entities must be considered. Since such a complete treatment was found to be prohibitive in terms of computer time a simplifying procedure due to Porile and Tanaka[19] was adopted. It consisted, briefly, of replacing all heavy particles with $A\leq10$ by Li^7 in its ground state and multiplying the emission probability of the latter by the average ratio of emission probabilities of all the mass 6 to 10 nuclei in particle bound states to that of Li^7 in its ground state. The value of this ratio was found to be 4.5 on the basis of several cascades in which all the emission probabilities were computed.

The velocity of the recoiling nucleus was computed at each step of the evaporation process in the manner described elsewhere.[19] The direction of recoil was determined at each step by the choice of two random numbers on the assumption of isotropic evaporation. At the end of the evaporation process the kinetic energy of those products lying within certain mass ranges and angular intervals was computed for comparison with experiment. In order to duplicate the experimental angular acceptance, only those recoils traveling in a direction such that their angle with respect to the beam axis, was between 74 and 106° were included. In order to improve the statistical accuracy of the calculation all products with $A=82$–84 were grouped for comparison with the Sr^{83} data while all products with $A=59$–67 were combined for comparison with the $Cu^{61,64}$ results. This procedure is justified by the slow variation of range with product mass. None of the residual nuclei from the cascade had sufficiently high excitation energies to lead to either Na^{24} or Sc^{43}.

The calculated energy spectra are plotted as histograms in Fig. 4. The calculated and experimental spectra have been normalized to the same area in order to facilitate comparison. It is seen that there is good agreement for Sr^{83} and $Cu^{61,64}$. The mean recoil energies agree to within 2 MeV and the shapes are also in good agreement. It should be mentioned that, because of the long evaporation chain required to form $Cu^{61,64}$, the calculated spectrum reflects primarily the momentum imparted in the evaporation step. For the case of Sr^{83} the evaporation chain is considerably shorter and therefore the momenta imparted in the cascade and evaporation steps are about equally important in affecting the calculated spectrum.

Although the cascade calculation does not predict events with sufficient excitation energy to lead to the

[20] I. Dostrovsky, Z. Fraenkel, and G. Friedlander, Phys. Rev. **116**, 683 (1959).
[21] A. G. W. Cameron, Can. J. Phys. **36**, 1040 (1958).
[22] I. Dostrovsky, Z. Fraenkel, and P. Rabinowitz, Phys. Rev. **118**, 791 (1960).

formation of $Sc^{43,44}$ or Na^{24}, it is of interest to compute energy spectra for these products on the assumption that they are residues of a spallation process. On the one hand, the experiments were performed with 2.9-GeV protons while the calculation is for 1.8-GeV protons. A calculation for 2.9-GeV protons would presumably include events with sufficiently high excitation energies to lead to the heavier of the products in question. On the other hand, Na^{24} and $Sc^{43,44}$ lie in the mass region where products from two-body breakup may be expected to predominate, and their spectra may be characteristic of this mechanism. It would therefore be of interest to compute a spectrum for these products by a cascade-evaporation calculation, to determine how sensitive the energy spectrum is to the reaction mechanism.

In order to compute a recoil spectrum for Na^{24} or $Sc^{43,44}$, we have approximated the distribution of residual nuclei from the cascade by a single nuclide having sufficient excitation energy to lead to the product in question. This is a reasonable approximation in view of the fact that the difference in mass between the possible evaporation progenitors is expected to be much smaller than the mass difference between the progenitor and the product in question. We have started with Y^{85} with an excitation energy of 1100 MeV for the Na^{24} calculation, and with Mo^{90} excited to 800 MeV for $Sc^{43,44}$. These particular starting nuclei were obtained by a suitable extrapolation of the relation between number of cascade nucleons and average excitation energy.[5] The transverse components of velocity of the starting nuclei were each set equal to 0.2 $(\text{MeV/amu})^{1/2}$, these values also being obtained by extrapolation of the results of Porile.[18] Since the calculated results are rather insensitive to the forward component of velocity imparted by the cascade, this quantity was set equal to zero. This procedure obviated the need to restrict the products to a small angular interval. The results for all products with $A=21$–27 and $A=41$–46 were grouped for comparison with Na^{24} and $Sc^{43,44}$, respectively.

The experimental and calculated spectra are compared in the lower half of Fig. 4. The calculated spectra are based on 500 evaporation runs in each case and are normalized to the same area as the experimental spectra. It is seen that the calculated $Sc^{43,44}$ spectrum is in good agreement with the experimental one, while the calculation for Na^{24} does not agree well with experiment. These computed spectra are insensitive to reasonable changes in the parameters of the starting nuclei. The evidence indicates, therefore, that $Sc^{43,44}$ is formed from silver mostly by a spallation mechanism, while Na^{24} is formed by a splitting into two large fragments. Further experimental evidence for this conclusion is shown in Fig. 5. Here the observed range distributions have been transformed to velocity spectra and a comparison is made with the gross "fission fragment" velocity distribution as measured[12] for Ag and Br in nuclear emulsion. There is approximate agreement only with the Na^{24} spectrum. The selection criteria used in the emulsion work (range ratio ≤ 2) tend to reject lower velocity fragments and to narrow the velocity spectrum. Correction for this effect would improve the agreement with the Na^{24} velocity distribution. It appears, therefore, that the most abundant "two-body breakup" products formed from silver irradiated by 2.9-GeV protons are in the mass 20–30 region.[23]

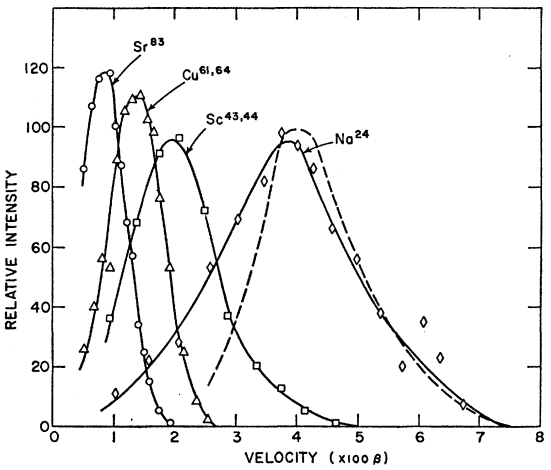

FIG. 5. Velocity spectra at 90° of products from 2.9-GeV proton irradiation of silver. The dashed curve shows the velocity distribution of "fission products" of Ag and Br observed in nuclear emulsion (Ref. 12).

For each species studied we can calculate what its Coulomb energy would be if it were formed by a mechanism in which the parent nucleus splits into two fragments. We assume that at some stage in the scission the two fragments are spheres in contact with the nuclear radius parameter $r_0=1.44$ F. As an extreme limit, we also assume that the parent species are the initial silver nuclei. The resulting Coulomb energies are shown in Fig. 4 by the arrows labeled C–1. Clearly, the observed mean energies are far below these values in every case. It may be significant, however, that for Na^{24} the ratio of calculated Coulomb energy to observed mean energy is only 2.3 whereas for $Sc^{43,44}$ and $Cu^{61,64}$ the corresponding ratios are 4.6 and 4.9, respectively. As was pointed out above, Na^{24} is the only one of the measured species which really appears to arise mainly by a two-body breakup mechanism.

The low mean energy of Na^{24} and the very broad energy spectrum can be explained in a qualitative way. The parent nuclei are most probably considerably lighter than silver. In fact the nuclear emulsion data[12] indicate that the nuclei which undergo scission are near the end of the evaporation phase. Furthermore, account

[23] In Ref. 12 it was estimated that "the most probable fission events yield products in the neighborhood of mass 35." However, that estimate was made without benefit of the additional radiochemical data reported here.

must be taken of all the recoil momenta from the initial cascade and from particle evaporation both before and after scission. A few typical paths for producing Na^{24} were tested by Monte Carlo calculation. Energies were found to be in rough agreement with the observed mean, and the values are not very sensitive to whether the scission takes place near the beginning or near the end of the evaporation phase. The wide distribution of observed energies probably cannot be accounted for only by the different ways in which the various velocity vectors couple with each other. Another important factor may be the variability of nuclear deformation prior to scission, even for the formation of the same products. Relatively minor broadening of the energy spectra arises from the circumstance that each product may result from a variety of parent nuclei and by β decay of various precursors.

We can now see why one should not expect a very substantial difference between the energy spectrum for a given product formed by the spallation mechanism and the spectrum for that product formed by a two body breakup mechanism. In the first case, the velocity imparted to the nucleus during the prompt cascade followed by evaporation of nucleons, alpha particles, and heavier particles, serve to displace the spectrum to higher energies. In the second case, the spectrum is displaced to lower energies from the nominal Coulomb energy by the processes discussed above.

The data of the present experiment are insufficient in themselves to determine whether the Na^{24} is formed from silver by a fast fragmentation mechanism or by a slower fission type mechanism. The fast mechanism is supported by the thick-target recoil experiments of Crespo, Alexander, and Hyde[3] while the slow mechanism is supported by the nuclear emulsion data of Baker and Katcoff.[12] It would be interesting to extend the present experiment for Na^{24} by making range measurements at forward and backward angles also, as was done recently by Cumming *et al.*[4] for Na^{24} produced from Bi. Their experiments showed that in this instance the Na^{24} was produced by a rapid process.

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Comparison Between Reactions of Alpha Particles With Scandium-45 and Deuterons with Titanium-47†

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The excitation functions for the (α,n), $(\alpha,2n)$, $(\alpha,2p)$, $(\alpha,\alpha'n)^m$, $(\alpha,\alpha'n)^g$, and $(\alpha,2pn)$ reactions of Sc^{45} and the (d,n), $(d,2n)$, $(d,2p)$, $(d,\alpha n)^m$, and $(d,\alpha n)^g$ reactions of Ti^{47} were measured for alpha-particle energies from 15 to 40 MeV and for deuteron energies from 4 to 20 MeV. The alpha-particle excitation functions can be successfully fitted by a calculation based upon compound-nucleus theory; but the calculation is less successful in reproducing the deuteron excitation functions. The divergences between calculation and observation are in a direction that is to be expected from the contribution of deuteron stripping reactions. Despite the indications that the deuteron-induced reactions have substantial noncompound contributions, the ratio of the cross sections for the $(\alpha,2p)$ and $(\alpha,2n)$ reactions have the same dependence upon the excitation energy of the compound system as has the corresponding ratio for the $(d,2p)$ and $(d,2n)$ reactions.

I. INTRODUCTION

MANY investigations have been carried out on nuclear reactions induced by protons and alpha particles in the medium weight elements ($45<A<75$) at energies up to a few tens of MeV. Dostrovsky *et al.*[1] have succeeded, to a large extent, in fitting all the existing excitation functions with the compound-nucleus model. Implicit in this model is the assumption made by Bohr in 1937 that the modes of decay of the compound nucleus are independent of the modes of formation. To

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[1] I. Dostrovsky, Z. Fraenkel, and G. Friedlander, Phys. Rev. 116, 683 (1959).